Vectors – Basics

1. $\mathbf{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

(a) Sketch the vectors **p** and **q.**
(b) Sketch the vectors –**p** and –**q.**
(c) Given $\mathbf{u} = \mathbf{p} + \mathbf{q}$, sketch the vector **u.**
(d) Given $\mathbf{v} = \mathbf{p} - \mathbf{q}$, sketch the vector **v.**

2. $\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -7 \\ -4 \end{pmatrix}$

(a) Sketch the vectors **a** and **b.**
(b)Given $\mathbf{u} = \mathbf{a} + \mathbf{b}$, sketch the vector **u.**
(c)Given $\mathbf{v} = \mathbf{a} - \mathbf{b}$, sketch the vector **v.**

3. $\mathbf{a} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 6 \\ -8 \\ 0 \end{pmatrix}$.

Find (a) 3**a** (b) –**b** (c) 2**a** + 3**b** (d) 4**b** – **a** (e) -½**b**

(f) $|\mathbf{b}|$ (g) $|2\mathbf{a}|$ (h) $|\mathbf{b} + 2\mathbf{a}|$

4. Find **p** and **q** in each equation below

(a) $\begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$ (c) $\begin{pmatrix} -3 \\ 1 \end{pmatrix} - \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} = 3\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

5. Find **p**, **q** and **r** in each of the following

(a) $\begin{pmatrix} \mathbf{p} \\ \mathbf{q} \\ \mathbf{r} \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} \mathbf{p} \\ \mathbf{q} \\ \mathbf{r} \end{pmatrix} + 2\begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$

6. A is the point (2,1,-2) and B is the point (0,-3,4).

(a) Write down the components of **a** and **b**, the position vectors of A and B.
(b) Calculate (i) –2**a** + **b** (ii) $|-2\mathbf{a} + \mathbf{b}|$
(c) Find the vector $\overrightarrow{AB}$

7. P is the point (1,-1,2), Q is (2,0,-5) and R is (1,1,0).

(a) Write down the components of **p**, **q** and **r** the position vectors of P, Q and R.
(b) Find the vectors (i) $\overrightarrow{QP}$ (ii) $\overrightarrow{QR}$ (iii) $\overrightarrow{PR}$

8. M is the point (2,3,-5), N is (1,1,0) and R is (-4,2,-2).

(a) Write down the components of **m**, **n** and **r** the position vectors of M, N and R.

(b) Find the vectors (i) $\overrightarrow{MN}$ (ii) $\overrightarrow{RN}$ (iii) $\overrightarrow{MR}$

9. PQRS is a parallelogram with vertices P(3,4,0), Q(7,6,-3) and R(8,5,2). Find the coordinates of S. (Hint: $\overrightarrow{PQ} = \overrightarrow{SR}$).

10. A is the point (2,1,-6), B is (3,1,-9) and C is (0,1,6). Given $\overrightarrow{AD} = \frac{2}{3}\overrightarrow{BC}$, find the coordinates of D.

11. P is the point (-4,2,2), Q is (-1,8,14) and R is (-5,2,10). Given $\overrightarrow{PQ} = \frac{3}{4}\overrightarrow{RS}$, find the coordinates of S.

12. (a) Calculate the magnitude of the vector $\mathbf{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(b) Find a unit vector parallel to the vector **u**.

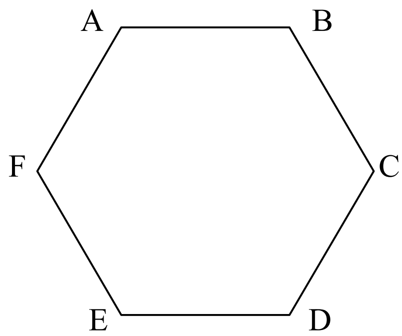
13. (a) Calculate the magnitude of the vector $\mathbf{w} = \begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix}$

(b) Find a unit vector parallel to **w**.

14. (a) Calculate $|\mathbf{a}|$ where $\mathbf{a} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$

(b) Find a unit vector parallel to **a**.

15. The diagram shows a regular hexagon.



(i) Write down another vector equal to

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{CD}$

(ii) Find a vector equal to

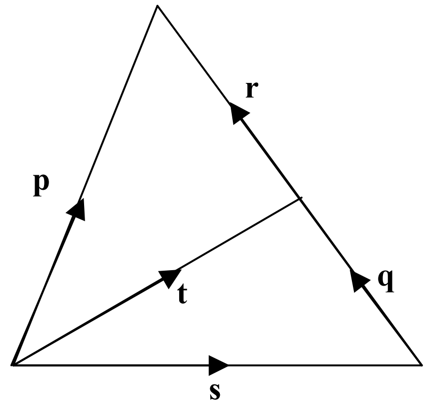
(a) $\overrightarrow{AB} + \overrightarrow{BC}$ (b) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ (c) $\overrightarrow{FE} + \overrightarrow{ED}$

(d) $\overrightarrow{FA} + \overrightarrow{ED}$ (e) $\overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{EF}$ (f) $\overrightarrow{BC} - \overrightarrow{DC}$

16. Use the diagram opposite to name the vector that represents

(a) $\mathbf{p} - \mathbf{r}$ (b) $\mathbf{r} - \mathbf{p}$ (c) $\mathbf{t} - \mathbf{q}$ (d) $\mathbf{s} - \mathbf{t}$

(e) $\mathbf{p} - \mathbf{r} - \mathbf{q}$ (f) $\mathbf{t} + \mathbf{r} - \mathbf{p}$ (g) $\mathbf{s} - \mathbf{p} + \mathbf{q}$

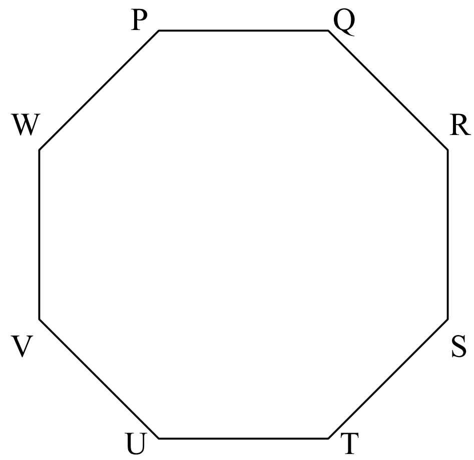


17. The diagram shows a regular octagon.

(a) Write down another vector equal to

(i) $\overrightarrow{WP}$ (ii) $\overrightarrow{WV} + \overrightarrow{VU}$

(iii) $\overrightarrow{WP} + \overrightarrow{SR} + \overrightarrow{RQ}$ (iv) $\overrightarrow{SR} - \overrightarrow{ST}$



18. The diagram shows a cuboid ABCD EFGH.

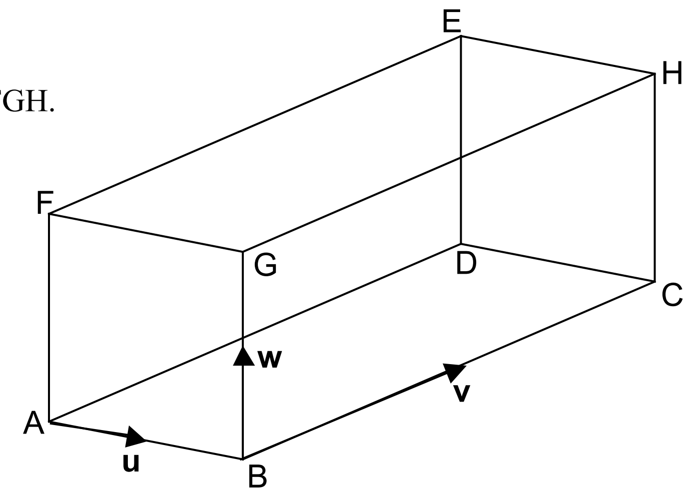
(a) Express in terms of **u**, **v** and **w**.

(i) $\overrightarrow{DC}$ (ii) $\overrightarrow{ED}$

(iii) $\overrightarrow{FH}$ (iv) $\overrightarrow{HA}$

(b) Express in terms of ABCD EFGH

(i) $\mathbf{u} + \mathbf{v}$ (ii) $\mathbf{u} - \mathbf{w}$ (iii) $\mathbf{w} - \mathbf{u} + \mathbf{v}$



19. In the trapezium AB = 2DC and AB is parallel to DC.

In terms of **u** and **v**, write down the vectors

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{AC}$ (c) $\overrightarrow{BC}$ (d) $\overrightarrow{AN}$