What is a Vector ?

There are two types of measurable quantities in this world :-

SCALARS and **VECTORS**.

Definition :- A **scalar** quantity is one that only requires **size** (or **magnitude**) to define it fully.
It does **not** require a sense of "**direction**" to be assigned to it.

Examples :-

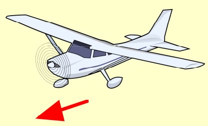
Time	-	3 seconds, 5 minutes, 4 days, a year.
Length	-	8 cm, 25 metres, 2·7 kilometres, half a mile.
Area	-	35 mm^2, 17 cm^2, 250 m^2, 3 acres.
Speed	-	160 km/hr, 3500 mph, 25 cm per sec.

Definition :- A **vector** quantity is one that requires, not just **magnitude** (or **size**) to define it, but needs an indication of its **direction**.

Examples :-

Displacement - (movement) : If you want to describe a walk you have just done, it is not good enough to just tell how far you have gone.
You have to tell where you went or in other words, what **direction** you took.

Velocity - A pilot would be in trouble if all he gave to air traffic control was what speed he was doing. He would have to tell them his speed **and** in what **direction** he was travelling.



Force - When a force is applied to slide a box of matches, just knowing the "strength" of the force is not enough. You won't be able to tell where the box ends up without being told in what **direction** the force was being applied.

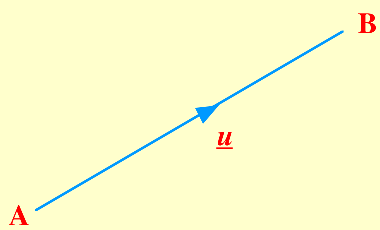
Representing a Vector

We can represent a vector by what is called a **directed line segment**.

The diagram shows a **journey** from **A** to **B**, written as $\overrightarrow{AB}$ or $\underline{u}$.

(Some books use bold italics $\boldsymbol{u}$, but this is difficult for you to show).

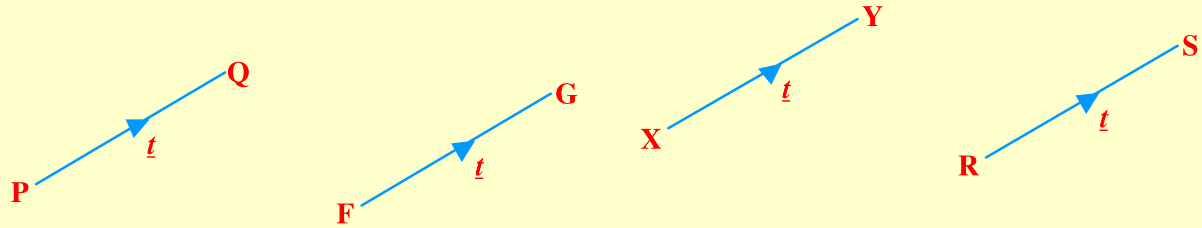
Instead, in this chapter, we will emphasise it is a vector by underlining it, thus representing it as the vector $\underline{u}$.



Note that :- The **magnitude** of the vector, or displacement, is represented by the length of the line.
The **direction** of the vector is shown by the arrow.

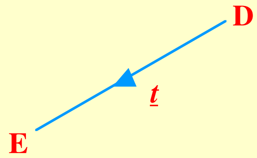
Equivalent Vectors

A vector journey simply tells you where you end up at, in relation to where you began your journey. For this reason, if two or more directed line segments have the **same length and the same direction**, then they represent the same **vector** (journey). These four vectors are all equivalent.



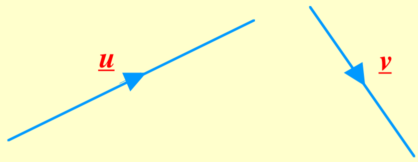
In other words :- $\overrightarrow{PQ} = \overrightarrow{FG} = \overrightarrow{XY} = \overrightarrow{RS} = \underline{t}$.

Can you also see why vector $\overrightarrow{DE} = -\underline{t}$? (*the opposite direction*).



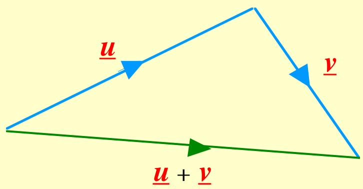
Adding Vectors

Shown are two vectors $\underline{u}$ and $\underline{v}$.



Imagine you took a journey along in the direction of vector $\underline{u}$, then turned and took a second journey in the direction $\underline{v}$.

To find the single equivalent vector journey, as far as your starting point and finishing point is concerned, you simply redraw the vectors and add them **nose** to **tail**.



This new **single** journey, equivalent to both combined journeys, is the vector :-

$$\underline{u} + \underline{v}.$$

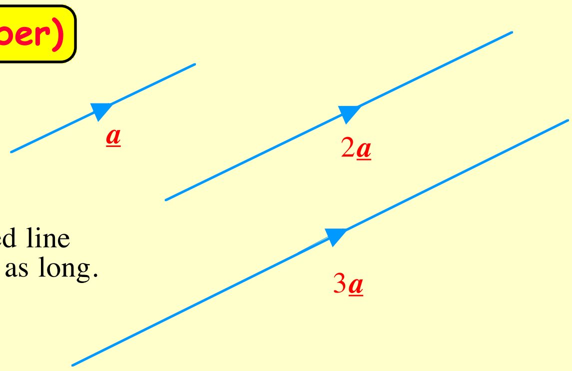
Multiplying a Vector by a Scalar (a number)

Shown this time is the vector $\underline{a}$.

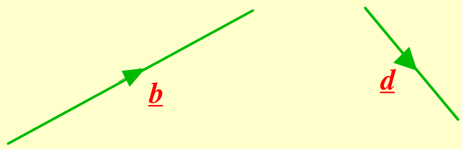
Imagine you took a journey in the same direction as vector $\underline{a}$, but travelled twice (or three times) as far.

This time, the new vector can be represented by a directed line segment, parallel to the vector, but twice (or three times) as long.

i.e. $2\underline{a}$ (or $3\underline{a}$).

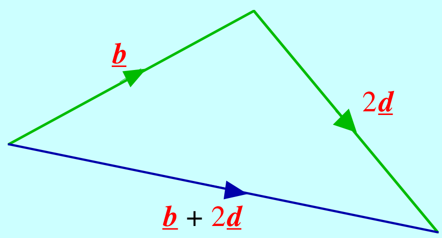


Example :- Given the two vectors $\underline{b}$ and $\underline{d}$, sketch them and show the vector $\underline{b} + 2\underline{d}$.



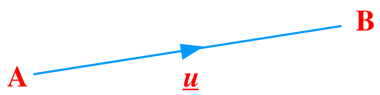
Solution :- Draw vector $\underline{b}$ first. Then, onto the end of $\underline{b}$, you simply add on a vector equivalent to, but twice the length of $\underline{d}$.

The **blue arrowed line** represents the vector $\underline{b} + 2\underline{d}$.

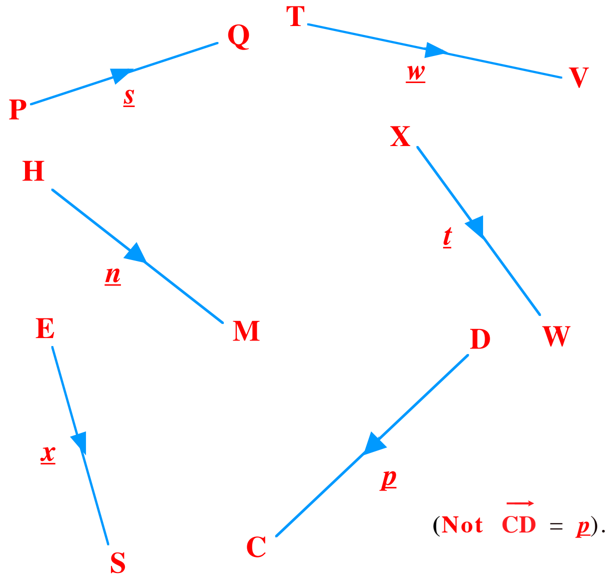


Exercise 13·1

1. This vector is given as :- $\overrightarrow{AB} = \underline{u}$.

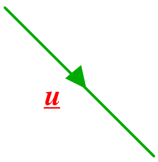


Name each of these vectors in **two** ways :-



(Not $\overrightarrow{CD} = \underline{p}$).

2. Sketch this vector $\underline{u}$.



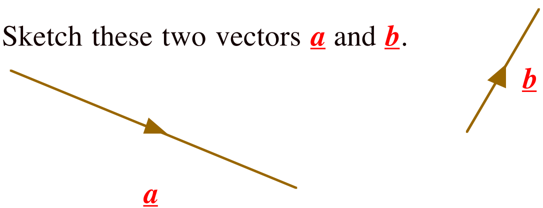
   (a) Now sketch the vector $2\underline{u}$.

   (b) Sketch the vector $3\underline{u}$.

   (c) Sketch the vector $-\underline{u}$.

   (d) Sketch the vector $-4\underline{u}$.

3. Sketch these two vectors $\underline{a}$ and $\underline{b}$.



3. (a) Now sketch the vector $2\underline{b}$.

   (b) Sketch the vector showing $\underline{a} + \underline{b}$ and mark $\underline{a} + \underline{b}$ on your diagram.

   (c) This time, sketch the vector $\underline{b} + \underline{a}$.
   (*i.e. start drawing* $\underline{b}$ *first, then* $\underline{a}$).

   (d) Do the vectors you have drawn in parts (b) and (c) look the same ?

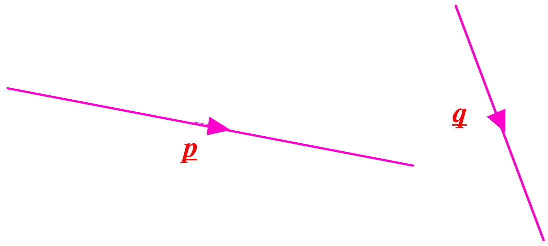
   (e) What does this tell you about how you draw vectors $\underline{a} + \underline{b}$ or $\underline{b} + \underline{a}$?

   (f) Similarly, sketch the vector $\underline{a} + 2\underline{b}$.

   (g) Sketch the vector $2\underline{a} + \underline{b}$.

   (h) Do the vectors $\underline{a} + 2\underline{b}$ and $2\underline{a} + \underline{b}$ look the same in your sketches ?

4. Sketch the vectors $\underline{p}$ and $\underline{q}$.



   (a) Sketch the vector $\underline{p} + \underline{q}$.

   (b) Now sketch an label vectors $-\underline{p}$ and $-\underline{q}$.

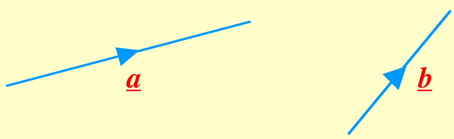
   (c) Sketch the vector $-(\underline{p} + \underline{q})$.
   (*This is the same as* $\underline{p} + \underline{q}$ *but in the opposite direction*).

   (d) From your sketches in part (b), show the vector $-\underline{p} + -\underline{q}$. Is it the same as $-(\underline{p} + \underline{q})$?

Subtracting Vectors

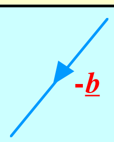
If you wish to **subtract** vectors such as $\underline{a} - \underline{b}$, it is easier to draw the vectors $\underline{a}$ and $-\underline{b}$, then **add**.

=> $\underline{a} - \underline{b} = \underline{a} + -\underline{b}$.

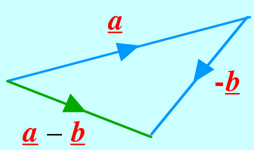


Example :-

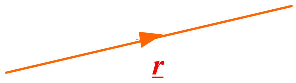
Draw $-\underline{b}$ first. =>

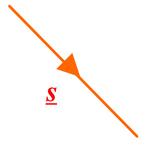


Then add $\underline{a} + -\underline{b}$ => $\underline{a} - \underline{b} = \underline{a} + -\underline{b}$ =>



5. Sketch the vectors $\underline{r}$ and $\underline{s}$.





(a) Sketch the vector $\underline{r} + \underline{s}$.

(b) Now sketch and label vector $\underline{r} - \underline{s}$.

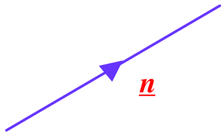
(c) Sketch the vector $\underline{s} - \underline{r}$. (*Draw* $\underline{s}$ first).

(d) Sketch the vector **$-2\underline{s}$**.

(e) Now sketch $\underline{r} - 2\underline{s}$.

(f) Sketch $3\underline{s} - 2\underline{r}$.

6. Shown is the vector $\underline{n}$.



(a) Sketch $\underline{n}$.

(b) Sketch $-\underline{n}$.

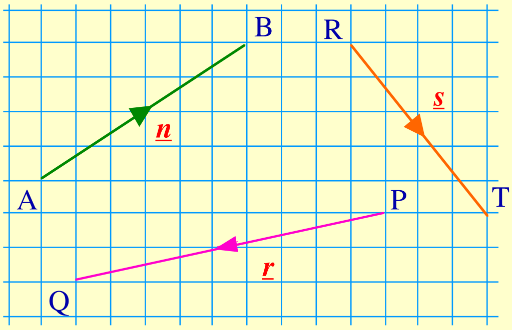
(c) Try to sketch the vector $\underline{n} + -\underline{n}$.

When you add a vector to its negative, you end up where you started. (*They cancel each other*).

This is referred to as the **zero vector**.
$\underline{u} + -\underline{u}$ is the same as $\underline{u} - \underline{u} = \underline{0}$. (*Note how its written*).

Representing Vectors in 2 Dimensions

We can introduce numerical values for vectors by representing them on a 2 dimensional grid as shown.



Vector $\overrightarrow{AB} = \underline{n}$ represents a journey, (a **translation**), from point A to point B, and this can be achieved by starting at A, moving **6 boxes right**, then **4 boxes up**, and arriving finally at point B.
We can represent this as follows :-

$\overrightarrow{AB} = \underline{n} = \begin{pmatrix}6\\4\end{pmatrix}$.

6 boxes **right**
followed by
4 boxes **up**

The **6** is referred to as the **horizontal component** of the vector.
The **4** is referred to as the **vertical component**.

In the diagram, two other vectors are shown, $\overrightarrow{RS}$ and $\overrightarrow{PQ}$.

To travel from point R to point S, you move **4 right** and **5 down**. => $\overrightarrow{RS} = \begin{pmatrix}4\\-5\end{pmatrix}$.

* **note** the negative component.

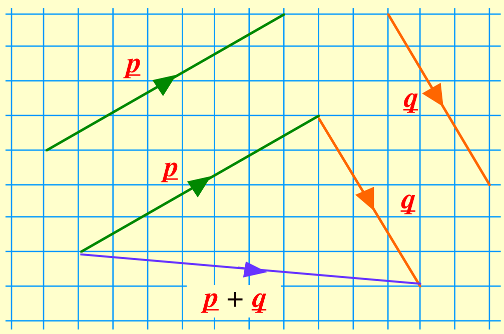
To travel from point P **back** to point Q, you move **9 left** and **2 down**. => $\overrightarrow{PQ} = \begin{pmatrix}-9\\-2\end{pmatrix}$.

* **note** both the components are negative.

Adding Vectors in 2 Dimensions

We have already seen that to add two vectors, you simply draw the first, then join the tail of the second onto the head of the first.

This is easily seen in two dimensions.



Shown are 2 vectors, $\underline{p} = \begin{pmatrix}7\\4\end{pmatrix}$ and $\underline{q} = \begin{pmatrix}3\\-5\end{pmatrix}$.

To find what vector $\underline{p} + \underline{q}$ looks like, we add.

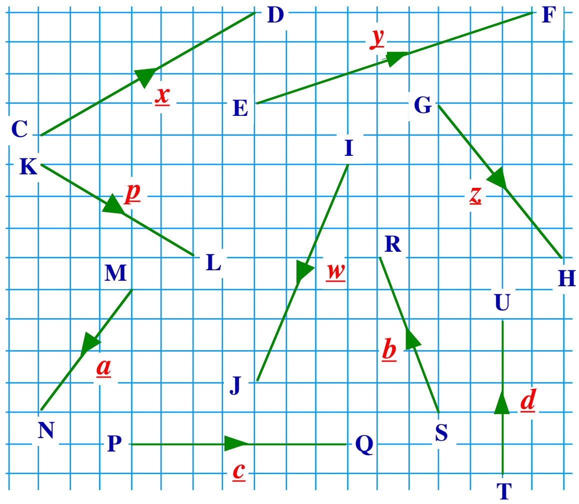
Note that :- $\underline{p} + \underline{q} = \begin{pmatrix}7\\4\end{pmatrix} + \begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}7+3\\4+(-5)\end{pmatrix} = \begin{pmatrix}10\\-1\end{pmatrix}$

To **add** two vectors whose components are known, you simply **add** the corresponding **components**.

Exercise 13·2

1. Use brackets to write down the 2 dimensional components of the following vectors :-

Example :- $\overrightarrow{AB} = \underline{p} = \begin{pmatrix}4\\2\end{pmatrix}$.



2. On squared paper, draw and label representatives of the following vectors :-

(a) $\underline{p} = \begin{pmatrix}4\\2\end{pmatrix}$ (b) $\underline{q} = \begin{pmatrix}5\\3\end{pmatrix}$

(c) $\underline{r} = \begin{pmatrix}4\\-2\end{pmatrix}$ (d) $\underline{s} = \begin{pmatrix}1\\-5\end{pmatrix}$

(e) $\overrightarrow{AB} = \begin{pmatrix}-2\\3\end{pmatrix}$ (f) $\overrightarrow{HK} = \begin{pmatrix}-5\\0\end{pmatrix}$

(g) $\overrightarrow{UV} = \begin{pmatrix}-8\\-3\end{pmatrix}$ (h) $\overrightarrow{ST} = \begin{pmatrix}0\\-6\end{pmatrix}$.

3. (a) On squared paper, draw the vector $\underline{a} = \begin{pmatrix}3\\2\end{pmatrix}$.

(b) Beside it, draw the vector $2\underline{a}$.

3. (c) Write down the **components** of vector $2\underline{a}$.

Can you see that $2\underline{a} = \begin{pmatrix}6\\4\end{pmatrix} = 2 \times \begin{pmatrix}3\\2\end{pmatrix} = 2 \times \underline{a}$?

> If you multiply a vector $\underline{a}$ by a number (a scalar), you simply multiply each **component** of the vector by that number.

In question 4, you are not required to draw the vectors. However, if you feel it would help, please feel free to do so.

4. Given $\underline{r} = \begin{pmatrix}6\\2\end{pmatrix}$ and $\underline{s} = \begin{pmatrix}4\\-3\end{pmatrix}$, find :-

(a) $\underline{r} + \underline{s}$ (b) $\underline{r} - \underline{s}$

(c) $2\underline{r}$ (d) $3\underline{s}$

(e) $2\underline{r} + 3\underline{s}$ (f) $3\underline{s} - 2\underline{r}$

(g) $4\underline{r} - \underline{s}$ (h) $-\underline{r}$

(i) $\underline{r} + -\underline{r}$ (j) $\underline{s} - \underline{s}$.

5. Draw vectors $\underline{a} = \begin{pmatrix}5\\2\end{pmatrix}$, $\underline{b} = \begin{pmatrix}-1\\-4\end{pmatrix}$ and $\underline{c} = \begin{pmatrix}-4\\2\end{pmatrix}$.

(a) On your grid show how to add $\underline{a} + \underline{b}$.

(b) Check from your drawing :- $\underline{a} + \underline{b} = \begin{pmatrix}4\\-2\end{pmatrix}$.

(c) Without the aid of a drawing, find $\underline{a} + \underline{b} + \underline{c}$.

(d) Explain your answer.

6. Solve these **vector equations** for vector $\underline{x}$.

(a) $\underline{x} + \begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}7\\9\end{pmatrix}$ (b) $\underline{x} - \begin{pmatrix}2\\8\end{pmatrix} = \begin{pmatrix}5\\-3\end{pmatrix}$

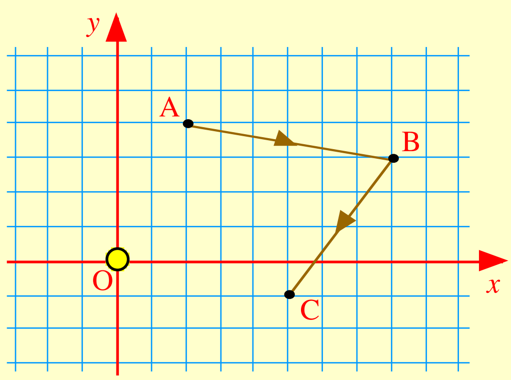
(c) $2\underline{x} = \begin{pmatrix}10\\-6\end{pmatrix}$ (d) $5\underline{x} = \begin{pmatrix}200\\-80\end{pmatrix}$

(e) $3\underline{x} - \begin{pmatrix}4\\-1\end{pmatrix} = \begin{pmatrix}11\\7\end{pmatrix}$ (f) $6\underline{x} - \begin{pmatrix}2\\5\end{pmatrix} = 2\underline{x} + \begin{pmatrix}10\\-1\end{pmatrix}$.

Vectors represented in a Coordinate Diagram

An obvious place to represent vectors is in a Cartesian Diagram, with the positions of points being given in terms of a fixed point O, the **origin**.

Shown are 3 points A(2, 4), B(8, 3) and C(5, -1).



- Check that vector $\overrightarrow{AB} = \begin{pmatrix}6\\-1\end{pmatrix}$ and vector $\overrightarrow{BC} = \begin{pmatrix}-3\\-4\end{pmatrix}$.
- Check both diagrammatically, and using components that,

  by adding the two vectors, we get $\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix}3\\-5\end{pmatrix} = \overrightarrow{AC}$.
- Check both diagrammatically, and using components that,

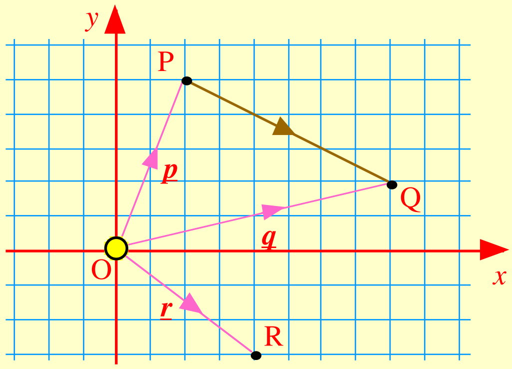
  by adding the vectors, we get $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \begin{pmatrix}0\\0\end{pmatrix}$, where $\overrightarrow{CA} = \begin{pmatrix}-3\\5\end{pmatrix}$.

Position Vectors

It is often handy to record the position of a point **in relation to the origin** by using **components**.

Here, the points P and Q are given by P(2, 5) and Q(8, 2).

Can you see that vector $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and vector $\overrightarrow{OQ} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$?



> Vectors like $\overrightarrow{OP}$, that define the **position** of a point in relation to the **origin** are defined by the vector $\underline{p}$.

Similarly, **position vector** $\overrightarrow{OQ} = \underline{q} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ and **position vector** $\overrightarrow{OR} = \underline{r} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

If you know the coordinates of two points P and Q, it is easy to determine the components of the vector joining them, without having to draw up a diagram.

In the above diagram, can you see that, in vector terms, the journey from P to Q can be completed by leaving P, going to the origin O, then going from O to Q ?

In vector terms, this means :- $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\underline{p} + \underline{q}$ or by reversing these, $= \underline{q} - \underline{p}$.

=> $\overrightarrow{PQ} = \underline{q} - \underline{p}$.

** This is a very important technique for determining vector components and should be learned.*

i.e. to find the vector $\overrightarrow{PQ}$, joining two points P and Q, you simply subtract the position vector $\underline{p}$ of the first point from the position vector $\underline{q}$ of the second point.

Example :- Determine the components of the vector $\overrightarrow{QR}$ in the above diagram.

Solution :- Vector $\overrightarrow{QR} = \underline{r} - \underline{q} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} - \begin{pmatrix} 8 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$. *(Check this out from the figure above).*

Exercise 13·3

1. (a) Plot the 2 points A(4, 1) and B(6, 8).
   (b) Use your diagram to write down the components of vector $\overrightarrow{AB}$.
   (c) Write down the components of the position vectors $\underline{a}$ and $\underline{b}$. (i.e. $\overrightarrow{OA}$ and $\overrightarrow{OB}$).
   (d) Find vector $\overrightarrow{AB}$, using $\overrightarrow{AB} = \underline{b} - \underline{a}$.

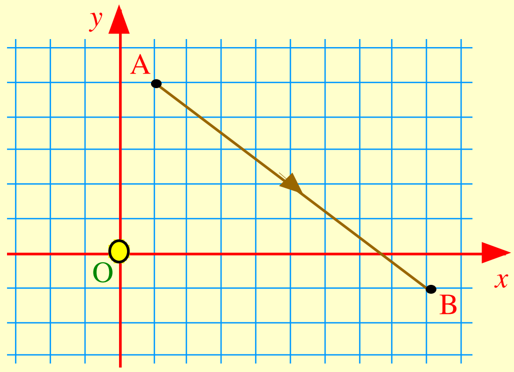
2. For each of the following pairs of points, find the components of the vector joining the first point to the second :-
   (a) U(4, 1), V(9, 3) i.e. find vector $\overrightarrow{UV}$.
   (b) S(0, 5), T(7, 2) (c) J(2, -3), K(5, 6)
   (d) P(-3, -4), Q(6, 0) (e) C(6, 2), D(1, -4)
   (f) G(2, -5), H(-1, 3) (g) A(-2, 7), B(-4, -5).

3. The coordinates of 6 points are P(1, 1), Q(5, 4), A(3, -2), B(7, 1), U(-4, 2) and V(0, 5).
   (a) Find the vector $\overrightarrow{PQ}$. ($\underline{q} - \underline{p}$).
   (b) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{UV}$.
   (c) What does this tell you about the three lines, PQ, AB and UV ?

4. M(1, -3), N(2, 1), R(4, -3) and S(6, 5).
   (a) Find the vectors $\overrightarrow{MN}$ and $\overrightarrow{RS}$.
   (b) Describe clearly the connection(s) between the lines MN and RS.

5. A(-1, -2), B(2, 4), C(7, 6) and D(4, 0).
   (a) Without plotting the points, find the components of the vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$.
   (b) What can you say about lines AB and DC ?
   (c) What type of quadrilateral does this fact tell you ABCD must be ?

The Magnitude of a Vector



A vector, (like displacement, velocity or force), requires direction as well as a sense of "size" to define it fully.

Sometimes we are only interested in the **size** of a vector, e.g. the **length** of the line, the **strength** of the force, or just the **speed** of an object. These are **scalar** quantities.

This is referred to as the **magnitude** of the vector, and when we are given a vector in component form, it is easy to calculate this, using **Pythagoras' Theorem**.

Given vector $\overrightarrow{AB}$ or $\underline{\boldsymbol{u}}$, the **magnitude** is denoted by :-

$|\overrightarrow{AB}|$ or $|\underline{\boldsymbol{u}}|$.

*(The "bars" either side,(the **modulus sign**), denotes the **magnitude** or "**size**" of the vector).*

Example :- The diagram shows two points A(1, 5) and B(9, -1). Find the length of the line AB.

Solution 1 :- One way of tackling the problem would be to draw a horizontal line from A and a vertical line from B to form a right angle triangle, count the horizontal and vertical number of boxes, then use **Pythagoras' Theorem** to determine the length of the sloping line AB.

A second, and more mathematical approach, is to use **vectors**.

The benefit of this is that you do not require a diagram to work with.

Solution 2 :- Step 1 :- Use A(1, 5) and B(9, -1) to find $\overrightarrow{AB} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}} = \begin{pmatrix} 9 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$.

The 8 and (–)6 are the lengths of the sides of the right angled triangle.

Step 2 :- Now use Pythagoras' as follows :- For **length**, think of **magnitude** instead.

$$|\overrightarrow{AB}| = \sqrt{8^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10.$$

In general, if vector $\overrightarrow{AB} = \underline{\boldsymbol{u}} = \begin{pmatrix} p \\ q \end{pmatrix}$, => **Magnitude** $|\overrightarrow{AB}| = |\underline{\boldsymbol{u}}| = \sqrt{p^2 + q^2}$.

Exercise 13·4

1. Calculate the distance from A(3, 1) to B(7, 4).
   Copy and **complete** :-

   $\overrightarrow{AB} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} ... \\ ... \end{pmatrix}$.

   $|\overrightarrow{AB}| = \sqrt{...^2 + ...^2} = \sqrt{......} = ...$

2. In a similar way, calculate the **magnitude**, (distance), between each pair of points here :-

   (a) U(4, 1), V(7, 5) (b) S(0, 5), T(12, 0)

   (c) J(2, -3), K(10, 3) (d) P(-3, -4), Q(6, 8)

   (e) C(6, 2), D(10, 2) (f) O(0, 0), H(15,8).

3. Not all square roots are **exact** of course.
   Calculate the distance from the two points P(5, 2) and Q(8, 5). (*See next column*).

3. **Copy** and **complete** :-

   $\overrightarrow{PQ} = \underline{\boldsymbol{q}} - \underline{\boldsymbol{p}} = \begin{pmatrix} 8 \\ 5 \end{pmatrix} - \begin{pmatrix} ... \\ ... \end{pmatrix} = \begin{pmatrix} ... \\ ... \end{pmatrix}$.

   $|\overrightarrow{PQ}| = \sqrt{...^2 + ...^2} = \sqrt{18} = 3\sqrt{2}$. *left in surd form.*

4. Calculate the **magnitude** of the vector joining these pairs of points, leaving your answer in **simplified surd** form where possible :-

   (a) I(5, 1), J(7, 5) (b) E(0, 5), F(8, 9)

   (c) M(1, -3), N(7, 3) (d) X(-3, -4), Y(5, 0)

   (e) S(5, 6), T(10, -4) (f) B(3, -1), C(6, 8).

5. S(2, -1), T(4, 3) and R(-2, 1) are 3 points.

   (a) Use the above method to calculate the lengths of the 3 sides of triangle STR, leaving your answers in **surd** form.

   (b) Use your answer to part (a) to explain clearly what kind of triangle STR is.

Exercise 13·5 (Mixed Examples)

1. Calculate the **magnitude** of the following vectors, leaving your answers in surd form :-

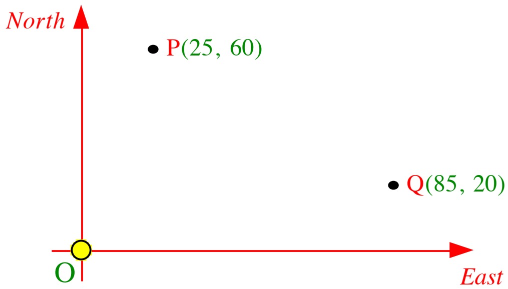
   (a) $\underline{u} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$, $|\underline{u}|$ =... (b) $\underline{s} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$, $|\underline{s}|$ =...

   (c) $\underline{a} = \begin{pmatrix} -9 \\ 3 \end{pmatrix}$, $|\underline{a}|$ =... (d) $\underline{v} = \begin{pmatrix} -4 \\ -8 \end{pmatrix}$, $|\underline{v}|$ =...

2. Given C(2, 3), P(8, 11), Q(10, -3) and R(-6, 9), show that P, Q and R could be points which lie on the circumference of a circle having its centre at point C.

   (*Hint : calculate the lengths of CP, CQ, ...*).

3. The coordinate diagram shows the position of two ships, the **P**latypus and the **Q**ueensway, in relation to **O**riskay harbour, (*distances in km*).



   (a) Describe, using components, the vector journey that the Platypus would have to travel to reach the Queensway.

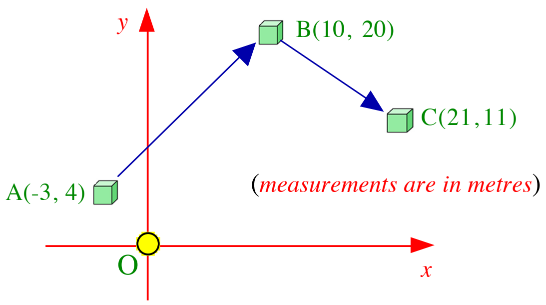
   (b) Use this to determine how far apart the two ships are at present, (*the magnitude*).

4. A force is applied to a large box in order to slide it from point A to point B.

   This force is represented by the vector $\overrightarrow{AB}$.

   A second force is then applied to move the box from point B to a new point C.

   This force is represented by the vector $\overrightarrow{BC}$.



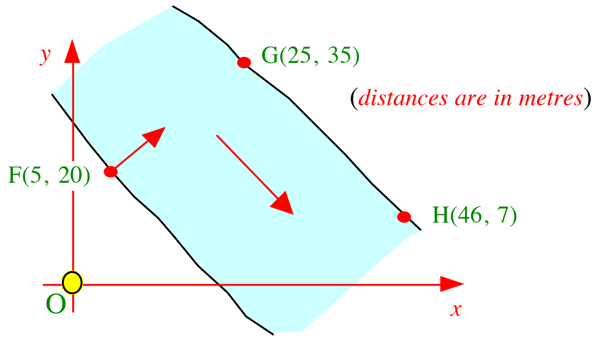
   (a) Find the components of forces $\overrightarrow{AB}$ & $\overrightarrow{BC}$.

   (b) Find the component of the **resultant** force (*i.e. the single force*), which if applied, would have moved the box directly from point A to point C.

   (c) Find the **magnitude** of this resultant force.

5. A boy attempts to swim across a river from point F to point G.

   Unfortunately, the current is forcing him downstream and he ends up at point H instead.



(*speed in metres/minute*)

   Vector $\overrightarrow{FG}$ represents the velocity of the journey he hoped to take him from F to G.

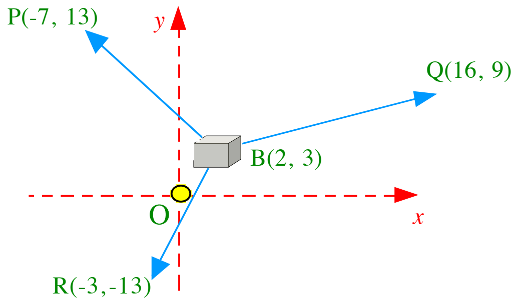
   Vector $\overrightarrow{GH}$ represents the velocity of the flowing stream that forces the swimmer to end up at H.

   (a) Find the components of $\overrightarrow{FG}$ and $\overrightarrow{GH}$.

   (b) Find $\overrightarrow{FH}$. This represents his actual journey.

   (c) Calculate the **speed** he was swimming at, the speed of the river and the **resultant** speed in his actual swim from F to H.

6. Three ropes are tied to a box and three boys pull the ropes in various directions as shown below.



   The coordinates of the box are (2, 3).

   The coordinates of the three points indicate, in relation to the box, the **strength** and **direction** of the force applied by each boy.

   (a) Determine the component values of the three forces, $\overrightarrow{BP}$, $\overrightarrow{BQ}$ and $\overrightarrow{BR}$.

   (b) Find the **magnitudes** of each force.

   (c) Add the 3 forces together. $\overrightarrow{BP} + \overrightarrow{BQ} + \overrightarrow{BR}$.

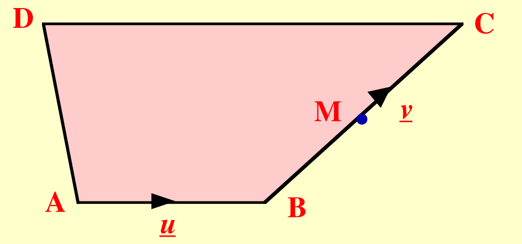
   (d) Explain your answer in terms of how, and in which direction the box actually moves.

Alternative Vector Journeys

As we said earlier, a displacement, (or vector), represents a journey from point A to point B.

As far as the vector is concerned, only the **finishing** point, **in relation to the starting** point, is important.

What **route** you take is **irrelevant**.



Examples :- This diagram above represents a **trapezium** with side DC equal to **2 ×** side AB in length.

Vector $\overrightarrow{AB} = \underline{u}$ and vector $\overrightarrow{BC} = \underline{v}$.

Find, in terms of $\underline{u}$ and $\underline{v}$, the following vectors :-

(a) $\overrightarrow{DC}$ **(b)** $\overrightarrow{AC}$ **(c)** $\overrightarrow{AD}$ **(d)** $\overrightarrow{BM}$ (*where M is the mid-point of BC*) **(e)** $\overrightarrow{DM}$.

Solutions :-

(a) $\overrightarrow{DC} = 2 \times \overrightarrow{AB}$ (since it is **parallel** to and **double** the length of AB) $= 2\underline{u}$.

(b) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \underline{u} + \underline{v}$.

* note the "–" sign

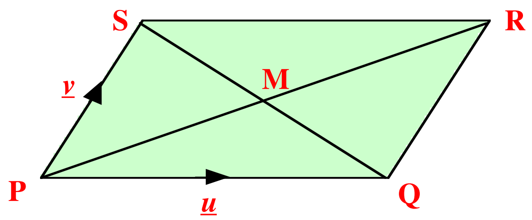
(c) $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \underline{u} + \underline{v} + -2\underline{u} = \underline{v} - \underline{u}$.

(d) $\overrightarrow{BM} = \frac{1}{2}$ of $\overrightarrow{BC} = \frac{1}{2}\underline{v}$.

(e) $\overrightarrow{DM} = \overrightarrow{DA} + \overrightarrow{AB} + \overrightarrow{BM} = -(\underline{v} - \underline{u}) + \underline{u} + \frac{1}{2}\underline{v} = 2\underline{u} - \frac{1}{2}\underline{v}$.

Exercise 13·6

1. Shown is parallelogram PQRS, with vector $\overrightarrow{PQ} = \underline{u}$ and vector $\overrightarrow{PS} = \underline{v}$.



Find the following vectors in terms of $\underline{u}$ and $\underline{v}$:-

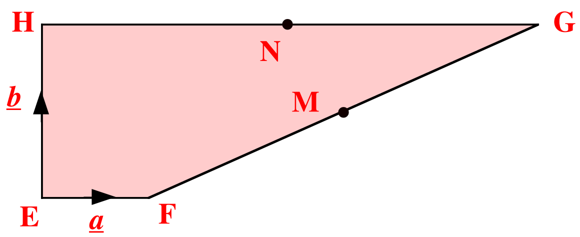
(a) $\overrightarrow{QR}$ (b) $\overrightarrow{SR}$ (c) $\overrightarrow{PR}$

(d) $\overrightarrow{QS}$ (e) $\overrightarrow{PM}$ (f) $\overrightarrow{SM}$.

2. The trapezium below has EF parallel to HG and HG = 4 × EF in length.

Vector $\overrightarrow{EF} = \underline{a}$ and vector $\overrightarrow{EH} = \underline{b}$.

M and N are the **mid-points** of FG and HG.



Find the following vectors in terms of $\underline{a}$ and $\underline{b}$:-

(a) $\overrightarrow{HG}$ (b) $\overrightarrow{EG}$ (c) $\overrightarrow{EN}$

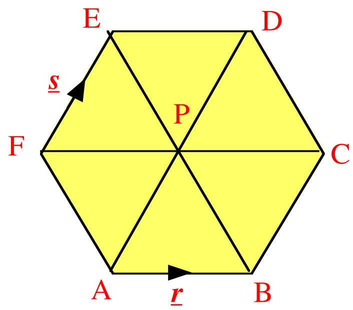
(d) $\overrightarrow{FG}$ (e) $\overrightarrow{GN}$ (f) $\overrightarrow{MN}$.

3. This time, ABCDEF is a hexagon with centre P.

Vector $\overrightarrow{AB} = \underline{r}$.

Vector $\overrightarrow{FE} = \underline{s}$.

Find the following in terms of $\underline{r}$ and $\underline{s}$:-

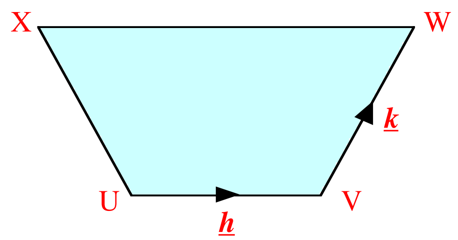


(a) $\overrightarrow{ED}$ (b) $\overrightarrow{BC}$

(c) $\overrightarrow{FC}$ (d) $\overrightarrow{AF}$

(e) $\overrightarrow{BE}$ (f) $\overrightarrow{AE}$.

4. Trapezium UVWX has UV parallel to WX and XW = 2 × UV in length. $\overrightarrow{UV} = \underline{h}$ and $\overrightarrow{VW} = \underline{k}$.



(a) Find these vectors in terms of $\underline{h}$ and $\underline{k}$:-

(i) $\overrightarrow{XW}$ (ii) $\overrightarrow{UW}$

(iii) $\overrightarrow{VX}$ (iv) $\overrightarrow{UX}$.

In fact, $\underline{h} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $\underline{k} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(b) Find the components of $\overrightarrow{XW}$, $\overrightarrow{UW}$ & $\overrightarrow{UX}$.

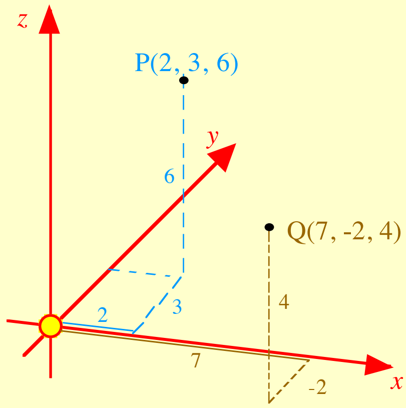
(c) Find $|\overrightarrow{VW}|$, $|\overrightarrow{UW}|$ & $|\overrightarrow{VX}|$ in surd form.

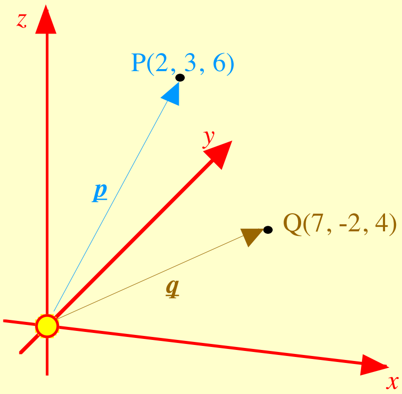
Vectors in 3 Dimensions

We saw, in an earlier chapter, how to represent a point in 3 dimensions using 3 coordinates instead of just 2.

The x and y-axes are lain in a **horizontal** plain and the z axis is **vertical** in relation to the other two axes.

Can you see that the 2 points are **P(2, 3, 6)** and **Q(7, -2, 4)**?





The **blue** arrow shows a 3-dimensional vector indicating the position of the point **P(2, 3, 6)**, from the origin.

The **brown** arrow shows a 3-dimensional vector giving the position of the point **Q(7, -2, 4)**, from the origin.

These two **position vectors** can be represented by :-

=> $\underline{p} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} 7 \\ -2 \\ 4 \end{pmatrix}$.
← the x component
← the y component
← the z component

As with in 2 dimensions, we can find the vector :-

$$\overrightarrow{PQ} = \underline{q} - \underline{p} = \begin{pmatrix} 7 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ -2 \end{pmatrix}.$$

Also, the **magnitude** can be found using **Pythagoras' Theorem**, (*applied in 3 dimensions*).

$$|\overrightarrow{OP}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

$$|\overrightarrow{PQ}| = |\underline{q} - \underline{p}| = \sqrt{5^2 + (-5)^2 + (-2)^2} = \sqrt{25 + 25 + 4} = \sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}.$$

Exercise 13·7

1. Given that $\underline{a} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$, find :-

   (a) $2\underline{a}$ (b) $3\underline{b}$ (c) $\underline{a} + \underline{b}$

   (d) $\underline{a} - \underline{b}$ (e) $2\underline{a} + 3\underline{b}$ (f) $-2(\underline{a} + \underline{b})$.

2. Given that $\underline{p} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} -3 \\ 4 \\ 12 \end{pmatrix}$, find :-

   (a) $\underline{p} + \underline{q}$ (b) $\underline{p} - \underline{q}$ (c) $-3\underline{p}$

   (d) $|\underline{p}|$ (e) $|\underline{q}|$ (f) $|\underline{p} + \underline{q}|$

   (g) Is it true that $|\underline{p}| + |\underline{q}| = |\underline{p} + \underline{q}|$?

3. Solve these **vector equations** for vector $\underline{x}$:-

   (a) $\underline{x} + \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ 1 \end{pmatrix}$ (b) $\underline{x} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -5 \end{pmatrix}$

3. (c) $2\underline{x} = \begin{pmatrix} 4 \\ -6 \\ 8 \end{pmatrix}$ (d) $-3\underline{x} = \begin{pmatrix} -9 \\ 12 \\ 0 \end{pmatrix}$

   (e) $4\underline{x} - \begin{pmatrix} 2 \\ 5 \\ 12 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix}$ (f) $5\underline{x} + \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 3\underline{x} + \begin{pmatrix} 9 \\ 1 \\ 6 \end{pmatrix}$.

4. P(1, 5, 8), Q(4, –1, 2) and R(6, 3, –4) are 3 points.

   (a) Write down the position vectors, $\underline{p}$, $\underline{q}$ and $\underline{r}$ of the 3 points P, Q and R. (i.e. $\overrightarrow{OP}$ etc.)

   (b) Using $\overrightarrow{PQ} = \underline{q} - \underline{p}$, find vector $\overrightarrow{PQ}$.

   (c) Similarly, find vectors $\overrightarrow{QP}$, $\overrightarrow{QR}$ and $\overrightarrow{RP}$.

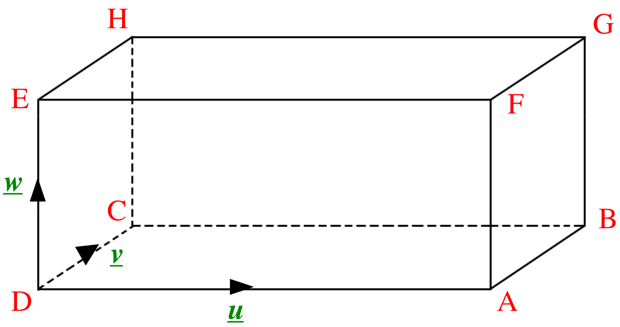
   (d) Find $\overrightarrow{PQ} + \overrightarrow{QP}$.

   (e) Explain your answer.

   (f) Now find $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP}$.

   (g) Explain this answer.

5. Shown is the cuboid ABCDEFGH. Sketch it.



$\overrightarrow{DA} = \underline{\boldsymbol{u}}$, $\overrightarrow{DC} = \underline{\boldsymbol{v}}$ and $\overrightarrow{DE} = \underline{\boldsymbol{w}}$.

Find, in terms of $\underline{\boldsymbol{u}}$, $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{w}}$, the vector :-

(a) $\overrightarrow{EF}$ (b) $\overrightarrow{AB}$ (c) $\overrightarrow{DF}$

(d) $\overrightarrow{DH}$ (e) $\overrightarrow{AG}$ (f) $\overrightarrow{DG}$

On your sketch, show the point R, the mid-point of AB, the point S, the middle of face ABGF and X at the very centre of the cuboid.

(g) $\overrightarrow{AR}$ (h) $\overrightarrow{DR}$ (i) $\overrightarrow{AS}$

(j) $\overrightarrow{DS}$ (k) $\overrightarrow{DX}$ (l) $\overrightarrow{HX}$.

6. In the above, $\underline{\boldsymbol{u}} = \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix}$, $\underline{\boldsymbol{v}} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$, and $\underline{\boldsymbol{w}} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$.

Find the following :-

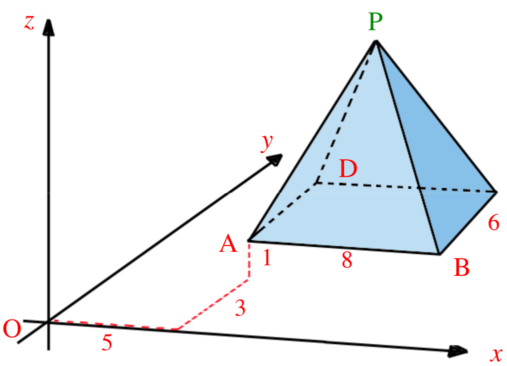
(a) $\overrightarrow{DH}$ (b) $\overrightarrow{DF}$ (c) $\overrightarrow{DG}$

(d) $|\overrightarrow{DH}|$ (e) $|\overrightarrow{DF}|$ (f) $|\overrightarrow{DX}|$.

7. Shown is a rectangular based pyramid with lengths 8 boxes and 6 boxes and with point P directly above the centre of rectangle ABCD.

AB is parallel to the *x*-axis.

The height of the pyramid is 12 boxes and the coordinates of point A are A(5, 3, 1).



(a) Write down the position vector, $\underline{\boldsymbol{a}}$, of A.

(b) Write down the position vectors of the other 4 points, B, C, D and P.

(c) Find vectors, $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AP}$.

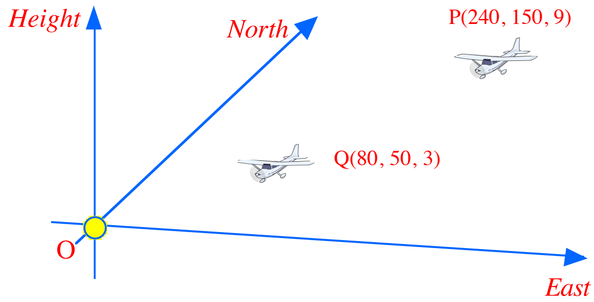
7. (d) Calculate the **magnitude** of the face diagonal vector $\overrightarrow{AC}$. (i.e. $|\overrightarrow{AC}|$).

(e) Calculate the **length** of AP. (i.e. $|\overrightarrow{AP}|$).

8. From the control tower (O) at an airport, the flight path of a small plane is being tracked. (*Distances are in kilometres*).

P shows where the plane is at 1500.

Q shows where the plane is at 1530.



(a) Write down the position vectors $\underline{\boldsymbol{p}}$ and $\underline{\boldsymbol{q}}$ of P and Q in relation to the control tower.

(b) Calculate how far away the plane is from the control tower at 1500 and 1530 (**magnitude**). (*Give each answer to the nearest kilometre*).

(c) Determine the components of the flight from P to Q. (i.e. $\overrightarrow{PQ}$).

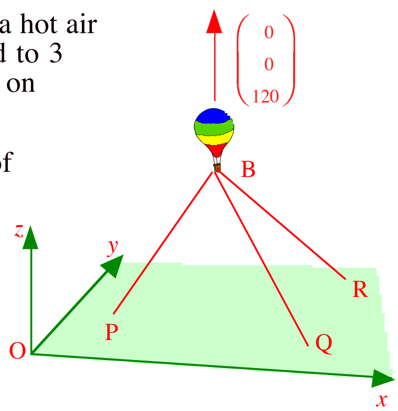
(d) By calculating $|\overrightarrow{PQ}|$, find the speed of the plane from P to Q.

(e) Explain why, if the plane keeps to its present flight path, it will arrive at the control tower.

9. The basket, B, of a hot air balloon is tethered to 3 points P, Q and R on the ground.

The coordinates of B, P, Q and R are given in relation to another point O.

The coordinates of all the points are given below.



P(8, 10, 0), Q(20, 5, 0), R(17, 30, 0), B(15, 15, 40).

The vectors $\overrightarrow{BP}$, $\overrightarrow{BQ}$ and $\overrightarrow{BR}$ represent the forces acting on the ropes holding the balloon.

The upwards arrow shows the vertical force acting on the balloon caused by the hot air.

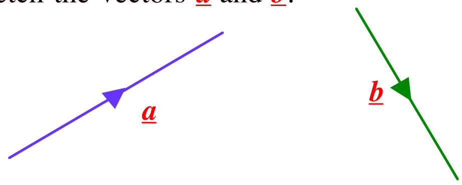
(a) Find the vectors $\overrightarrow{BP}$, $\overrightarrow{BQ}$ and $\overrightarrow{BR}$.

(b) By adding all 4 (force) vectors together, explain why the balloon remains in its fixed position.

Remember Remember..... ?

Topic in a Nutshell

1. Sketch the vectors $\underline{a}$ and $\underline{b}$.



(a) Sketch the vector $\underline{a} + \underline{b}$.

(b) Now sketch and label vector $\underline{a} - \underline{b}$.

(c) Sketch the vector $\underline{b} - \underline{a}$.

(d) Sketch the vector $-2\underline{a}$.

(e) Sketch $3\underline{b} - 2\underline{a}$.

2. Given $\underline{p} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$, find :-

(a) $\underline{p} + \underline{q}$ (b) $\underline{q} - \underline{p}$

(c) $3\underline{p}$ (d) $-2\underline{q}$

(e) $2\underline{p} + 3\underline{q}$ (f) $4\underline{q} - 2\underline{p}$.

3. Solve these **vector equations** for vector $\underline{x}$:-

(a) $\underline{x} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ (b) $\underline{x} - \begin{pmatrix} 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

(c) $2\underline{x} = \begin{pmatrix} 12 \\ -4 \end{pmatrix}$ (d) $7\underline{x} = \begin{pmatrix} -14 \\ 35 \end{pmatrix}$

(e) $4\underline{x} - \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$ (f) $5\underline{x} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} = 2\underline{x} + \begin{pmatrix} -7 \\ -1 \end{pmatrix}$

4. The coordinates of 4 points are :-

A(2, –3), B(8, 1), C(12, 1) and D(0, –7).

(a) Write the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ in component form.

(b) What does this tell you about the two lines lines, AB and CD ?

5. M is the point (–2, 7) and N is (3, –5).

Calculate $|\overrightarrow{MN}|$, the magnitude of MN.

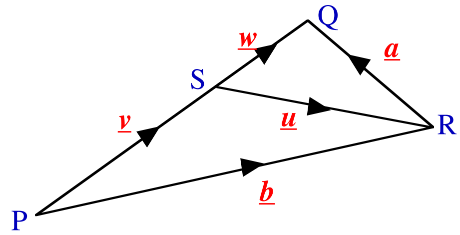
6. Given that $\underline{v} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$ and $\underline{w} = \begin{pmatrix} -4 \\ 3 \\ 12 \end{pmatrix}$, find :-

(a) $\underline{v} + \underline{w}$ (b) $\underline{v} - \underline{w}$ (c) $-2\underline{v}$

(d) $|\underline{v}|$ (e) $|\underline{w}|$

(f) Does $|\underline{v}| + |\underline{w}| = |\underline{v} + \underline{w}|$? Explain.

7. In the figure below, the directed line segments represent vectors as shown. For example ..... the line segment $\overrightarrow{PR}$ is represented by vector $\underline{b}$.



What line segment is represented by :-

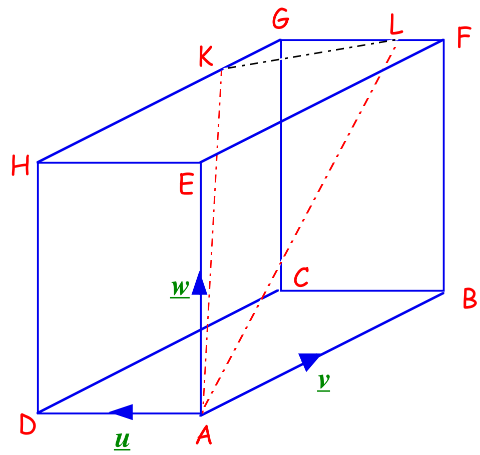
(a) vector $\underline{b} - \underline{u}$ (b) vector $\underline{w} - \underline{a}$

(c) vector $\underline{v} + \underline{u} - \underline{b}$ (d) vector $\underline{b} + \underline{a} - \underline{v} - \underline{w}$?

8. Solve these **vector equations** for vector $\underline{x}$:-

(a) $\underline{x} + \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$ (b) $2\underline{x} - \begin{pmatrix} -3 \\ 7 \\ -5 \end{pmatrix} = \begin{pmatrix} 11 \\ -9 \\ 17 \end{pmatrix}$.

9. ABCDHEFG is a cuboid.
K lies **two thirds** of the way along HG.
L lies **one quarter** of the way along FG.
$\overrightarrow{AD} = \underline{u}$, $\overrightarrow{AB} = \underline{v}$ and $\overrightarrow{AE} = \underline{w}$.

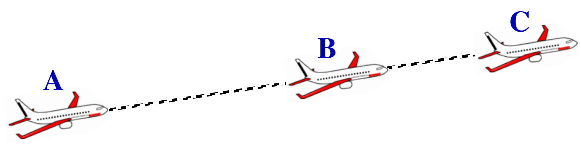


Find, in terms of $\underline{u}$, $\underline{v}$ and $\underline{w}$, the vector :-

(a) $\overrightarrow{FG}$ (b) $\overrightarrow{HG}$ (c) $\overrightarrow{LG}$

(d) $\overrightarrow{GK}$ (e) $\overrightarrow{AL}$ (f) $\overrightarrow{AK}$.

10.



An aircraft flying at a constant speed on a straight flight path takes 2 minutes to fly from A to B and one minute from B to C. Relative to a suitable set of axes, A is the point (–1, 3, 4) and B is (3, 1, –2).

Find the coordinates of point C.

Chapter 13 - Vectors

Exercise 13·1

1. (a) $\overrightarrow{PQ} = \underline{s}$ (b) $\overrightarrow{TV} = \underline{w}$ (c) $\overrightarrow{HM} = \underline{n}$
   (d) $\overrightarrow{XW} = \underline{t}$ (e) $\overrightarrow{ES} = \underline{x}$ (f) $\overrightarrow{DC} = \underline{p}$
2. see sketches
3. (a) - (c) see sketches (d) yes
   (e) you can add vectors in any order
   (f) - (g) see sketches (h) no
4. (a) - (d) see sketches
5. (a) - (f) see sketches
6. (a) - (b) see sketches
   (c) you get back to the start => total displacement = zero.

Exercise 13·2

1. (a) $\overrightarrow{CD} = \underline{x} = \begin{pmatrix}7\\4\end{pmatrix}$ (b) $\overrightarrow{EF} = \underline{y} = \begin{pmatrix}9\\3\end{pmatrix}$
   (c) $\overrightarrow{GH} = \underline{z} = \begin{pmatrix}4\\-5\end{pmatrix}$ (d) $\overrightarrow{KL} = \underline{p} = \begin{pmatrix}5\\-3\end{pmatrix}$
   (e) $\overrightarrow{IJ} = \underline{w} = \begin{pmatrix}-3\\-7\end{pmatrix}$ (f) $\overrightarrow{MN} = \underline{a} = \begin{pmatrix}-3\\-4\end{pmatrix}$
   (g) $\overrightarrow{SR} = \underline{b} = \begin{pmatrix}-2\\5\end{pmatrix}$ (h) $\overrightarrow{TU} = \underline{d} = \begin{pmatrix}0\\5\end{pmatrix}$
   (i) $\overrightarrow{PQ} = \underline{c} = \begin{pmatrix}7\\0\end{pmatrix}$
2. see sketches
3. (a) - (b) see sketches (c) $\begin{pmatrix}6\\4\end{pmatrix}$ - yes
4. (a) $\begin{pmatrix}10\\-1\end{pmatrix}$ (b) $\begin{pmatrix}2\\5\end{pmatrix}$ (c) $\begin{pmatrix}12\\4\end{pmatrix}$
   (d) $\begin{pmatrix}12\\-9\end{pmatrix}$ (e) $\begin{pmatrix}24\\-5\end{pmatrix}$ (f) $\begin{pmatrix}0\\-13\end{pmatrix}$
   (g) $\begin{pmatrix}20\\11\end{pmatrix}$ (h) $\begin{pmatrix}-6\\-2\end{pmatrix}$ (i) $\begin{pmatrix}0\\0\end{pmatrix}$
   (j) $\begin{pmatrix}0\\0\end{pmatrix}$
5. (a) see sketch (b) it does (c) $\begin{pmatrix}0\\0\end{pmatrix}$
   (d) you end back at the beginning again
6. (a) $\begin{pmatrix}5\\5\end{pmatrix}$ (b) $\begin{pmatrix}7\\5\end{pmatrix}$ (c) $\begin{pmatrix}5\\-3\end{pmatrix}$
   (d) $\begin{pmatrix}40\\-16\end{pmatrix}$ (e) $\begin{pmatrix}5\\2\end{pmatrix}$ (f) $\begin{pmatrix}3\\1\end{pmatrix}$

Exercise 13·3

1. (a) see sketch (b) $\begin{pmatrix}2\\7\end{pmatrix}$
   (c) $\begin{pmatrix}4\\1\end{pmatrix}$ and $\begin{pmatrix}6\\8\end{pmatrix}$ (d) $\begin{pmatrix}2\\7\end{pmatrix}$
2. (a) $\begin{pmatrix}5\\2\end{pmatrix}$ (b) $\begin{pmatrix}7\\-3\end{pmatrix}$ (c) $\begin{pmatrix}3\\9\end{pmatrix}$
   (d) $\begin{pmatrix}9\\4\end{pmatrix}$ (e) $\begin{pmatrix}-5\\-6\end{pmatrix}$ (f) $\begin{pmatrix}-3\\8\end{pmatrix}$
   (g) $\begin{pmatrix}-2\\-12\end{pmatrix}$
3. (a) $\begin{pmatrix}4\\3\end{pmatrix}$ (b) $\begin{pmatrix}4\\3\end{pmatrix}, \begin{pmatrix}4\\3\end{pmatrix}$ (c) parallel
4. (a) $\begin{pmatrix}1\\4\end{pmatrix}, \begin{pmatrix}2\\8\end{pmatrix}$ (b) parallel and twice the length
5. (a) $\begin{pmatrix}3\\6\end{pmatrix}, \begin{pmatrix}3\\6\end{pmatrix}$ (b) parallel & equal (c) parallelogram

Exercise 13·4

1. 5
2. (a) 5 (b) 13 (c) 10
   (d) 15 (e) 4 (f) 17
3. $3\sqrt{2}$
4. (a) $2\sqrt{5}$ (b) $4\sqrt{5}$ (c) $6\sqrt{2}$
   (d) $4\sqrt{5}$ (e) $5\sqrt{5}$ (f) $3\sqrt{10}$
5. (a) $2\sqrt{5}$ and $2\sqrt{5}$ and $2\sqrt{10}$ (b) Isosceles

Exercise 13·5

1. (a) $5\sqrt{2}$ (b) $3\sqrt{5}$
   (c) $3\sqrt{10}$ (d) $4\sqrt{5}$
2. Radius = 10 units
3 (a) $\begin{pmatrix}60\\-40\end{pmatrix}$ (b) $20\sqrt{13}$ km
4. (a) $\overrightarrow{AB} = \begin{pmatrix}13\\16\end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix}11\\-9\end{pmatrix}$ (b) $\begin{pmatrix}24\\7\end{pmatrix}$ (c) 25 m
5. (a) $\begin{pmatrix}20\\15\end{pmatrix}, \begin{pmatrix}21\\-28\end{pmatrix}$ (b) $\begin{pmatrix}41\\-13\end{pmatrix}$
   (c) 25 m/min, 35 m/min, 43 m/min
6. (a) $\begin{pmatrix}-9\\10\end{pmatrix}, \begin{pmatrix}14\\6\end{pmatrix}$ and $\begin{pmatrix}-5\\-16\end{pmatrix}$ (b) $\sqrt{181}, \sqrt{232}$ and $\sqrt{281}$
   (c) $\begin{pmatrix}0\\0\end{pmatrix}$ (d) resultant force is zero - no movement

Exercise 13·6

1. (a) $\underline{v}$ (b) $\underline{u}$ (c) $\underline{u} + \underline{v}$
   (d) $\underline{v} - \underline{u}$ (e) $1/2\underline{u} + 1/2\underline{v}$ (f) $1/2\underline{u} - 1/2\underline{v}$
2. (a) $4\underline{a}$ (b) $\underline{b} + 4\underline{a}$ (c) $\underline{b} + 2\underline{a}$
   (d) $\underline{b} + 3\underline{a}$ (e) $-2\underline{a}$ (f) $1/2\underline{b} - 1/2\underline{a}$
3. (a) $\underline{r}$ (b) $\underline{s}$ (c) $2\underline{r}$
   (d) $\underline{s} - \underline{r}$ (e) $2\underline{s} - 2\underline{r}$ (f) $2\underline{s} - \underline{r}$
4. (a) (i) $2\underline{h}$ (ii) $\underline{h} + \underline{k}$ (iii) $\underline{k} - 2\underline{h}$ (iv) $\underline{k} - \underline{h}$
   (b) $\begin{pmatrix}8\\0\end{pmatrix}, \begin{pmatrix}6\\4\end{pmatrix}$ and $\begin{pmatrix}-2\\4\end{pmatrix}$ (c) $2\sqrt{5}, 2\sqrt{13}$ and $2\sqrt{13}$

Exercise 13·7

1. (a) $\begin{pmatrix}8\\6\\-4\end{pmatrix}$ (b) $\begin{pmatrix}9\\-6\\15\end{pmatrix}$ (c) $\begin{pmatrix}7\\1\\3\end{pmatrix}$
   (d) $\begin{pmatrix}1\\5\\-7\end{pmatrix}$ (e) $\begin{pmatrix}17\\0\\11\end{pmatrix}$ (f) $\begin{pmatrix}-14\\-2\\-6\end{pmatrix}$

2. (a) $\begin{pmatrix}-2\\2\\14\end{pmatrix}$ (b) $\begin{pmatrix}4\\-6\\-10\end{pmatrix}$ (c) $\begin{pmatrix}-3\\6\\-6\end{pmatrix}$
(d) 3 (e) 13 (f) $2\sqrt{51}$
(g) no

3. (a) $\begin{pmatrix}6\\0\\-3\end{pmatrix}$ (b) $\begin{pmatrix}0\\0\\0\end{pmatrix}$ (c) $\begin{pmatrix}2\\-3\\4\end{pmatrix}$
(d) $\begin{pmatrix}3\\-4\\0\end{pmatrix}$ (e) $\begin{pmatrix}0\\2\\2\end{pmatrix}$ (f) $\begin{pmatrix}4\\-1\\2\end{pmatrix}$

4. (a) $\underline{p}=\begin{pmatrix}1\\5\\8\end{pmatrix}$, $\underline{q}=\begin{pmatrix}4\\-1\\2\end{pmatrix}$ and $\underline{r}=\begin{pmatrix}6\\3\\-4\end{pmatrix}$
(b) $\begin{pmatrix}3\\-6\\-6\end{pmatrix}$ (c) $\begin{pmatrix}-3\\6\\6\end{pmatrix}$, $\begin{pmatrix}2\\4\\-6\end{pmatrix}$ and $\begin{pmatrix}-5\\2\\12\end{pmatrix}$
(d) $\begin{pmatrix}0\\0\\0\end{pmatrix}$ (e) A vector + its negative gives zero
(f) $\begin{pmatrix}0\\0\\0\end{pmatrix}$
(g) Go round 3 sides of a triangle and you return to your starting point i.e. the zero vector.

5. (a) $\underline{u}$ (b) $\underline{v}$
(c) $\underline{u}+\underline{w}$ (d) $\underline{v}+\underline{w}$
(e) $\underline{v}+\underline{w}$ (f) $\underline{u}+\underline{v}+\underline{w}$
(g) $\frac{1}{2}\underline{v}$ (h) $\underline{u}+\frac{1}{2}\underline{v}$
(i) $\frac{1}{2}\underline{v}+\frac{1}{2}\underline{w}$ (j) $\underline{u}+\frac{1}{2}\underline{v}+\frac{1}{2}\underline{w}$
(k) $\frac{1}{2}\underline{u}+\frac{1}{2}\underline{v}+\frac{1}{2}\underline{w}$ (l) $-\frac{1}{2}\underline{v}-\frac{1}{2}\underline{w}+\frac{1}{2}\underline{u}$

6. (a) $\begin{pmatrix}0\\3\\4\end{pmatrix}$ (b) $\begin{pmatrix}12\\0\\4\end{pmatrix}$ (c) $\begin{pmatrix}12\\3\\4\end{pmatrix}$
(d) 5 (e) $4\sqrt{10}$ (f) $6\frac{1}{2}$

7. (a) $\begin{pmatrix}5\\3\\1\end{pmatrix}$ (b) $\begin{pmatrix}13\\3\\1\end{pmatrix}$, $\begin{pmatrix}13\\9\\1\end{pmatrix}$, $\begin{pmatrix}5\\9\\1\end{pmatrix}$ and $\begin{pmatrix}9\\6\\13\end{pmatrix}$
(c) $\begin{pmatrix}8\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\6\\0\end{pmatrix}$ and $\begin{pmatrix}4\\3\\12\end{pmatrix}$
(d) 10 (e) 13

8. (a) $\begin{pmatrix}240\\150\\9\end{pmatrix}$ and $\begin{pmatrix}80\\50\\3\end{pmatrix}$ (b) 283 km and 94 km
(c) $\begin{pmatrix}-160\\-100\\-6\end{pmatrix}$ (d) dist = 189 km, speed = 378 km/hr
(e) $\overrightarrow{OQ}=\frac{1}{3}\overrightarrow{OP}$ means their paths are parallel and heading for the point O).
Also, OQ + QP = OP means Q lies on the line of OP.

9. (a) $\begin{pmatrix}-7\\-5\\-40\end{pmatrix}$, $\begin{pmatrix}5\\-10\\-40\end{pmatrix}$ and $\begin{pmatrix}2\\15\\-40\end{pmatrix}$ (b) $\begin{pmatrix}0\\0\\0\end{pmatrix}$
All forces cancel out meaning balloon is stationary.

Remember, Remember

1. see sketches

2. (a) $\begin{pmatrix}2\\-3\end{pmatrix}$ (b) $\begin{pmatrix}-8\\-1\end{pmatrix}$ (c) $\begin{pmatrix}15\\-3\end{pmatrix}$
(d) $\begin{pmatrix}6\\4\end{pmatrix}$ (e) $\begin{pmatrix}1\\-8\end{pmatrix}$ (f) $\begin{pmatrix}-22\\-6\end{pmatrix}$

3. (a) $\begin{pmatrix}1\\-7\end{pmatrix}$ (b) $\begin{pmatrix}6\\4\end{pmatrix}$ (c) $\begin{pmatrix}6\\-2\end{pmatrix}$
(d) $\begin{pmatrix}-2\\5\end{pmatrix}$ (e) $\begin{pmatrix}2\\2\end{pmatrix}$ (f) $\begin{pmatrix}-2\\1\end{pmatrix}$

4. (a) $\begin{pmatrix}6\\4\end{pmatrix}$ and $\begin{pmatrix}-12\\-8\end{pmatrix}$
(b) $\overrightarrow{CD}=-2\overrightarrow{AB}$ means parallel and twice the length.

5. 13

6. (a) $\begin{pmatrix}-2\\-1\\16\end{pmatrix}$ (b) $\begin{pmatrix}6\\-7\\-8\end{pmatrix}$ (c) $\begin{pmatrix}-4\\8\\-8\end{pmatrix}$
(d) 6 (e) 13 (f) no
The sum of the lengths of any 2 sides of a triangle is always greater than the length of the 3rd side.

7. (a) $\overrightarrow{PS}$ $(=\underline{v})$ (b) $\overrightarrow{SR}$ $(=\underline{u})$
(c) $\overrightarrow{PP}=\underline{0}$ (d) $\overrightarrow{PP}=\underline{0}$

8. (a) $\begin{pmatrix}4\\5\\0\end{pmatrix}$ (b) $\begin{pmatrix}4\\-1\\6\end{pmatrix}$

9. (a) $\underline{u}$ (b) $\underline{v}$ (c) $\frac{3}{4}\underline{u}$
(d) $-\frac{1}{3}\underline{v}$ (e) $\underline{v}+\underline{w}+\frac{1}{4}\underline{u}$ (f) $\underline{u}+\underline{w}+\frac{2}{3}\underline{v}$

10. C(5, 0, –5)